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Gravito-inertial fields and the theory of a neutral particle

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Abstract. The dynamics of a neutral particle is examined from the viewpoint of the theory of gravito-inertial fields. In particular, the concept of mass and the equivalence of the inertial and gravitational mass of a body are analysed here in more detail, suggesting a special mass structure for a fundamental particle.

In close parallel with the treatment by Rohrlich of the Abraham–Lorentz theory of the electron, it is shown that the field equations lead to the correct relativistically invariant energy and momentum relationships.

1. Introduction

It has been shown in a previous publication (Coster and Shepanski 1969) that a body moving relative to the observer may be regarded as producing an *inertial* field of strength $\mathscr{I}$, in addition to the gravitational field $\mathscr{G}$. This inertial field is analogous to the magnetic field of a moving charge. The combined gravito-inertial field is then described by a set of (Maxwellian type) field equations:

$$\begin{aligned} &\text{(i)}\quad \nabla\times\mathscr{G} = \delta_0\frac{\partial\mathscr{I}}{\partial t} &&\text{(iii)}\quad \nabla\cdot\mathscr{G} = -\frac{1}{\alpha_0}\rho_{\mathrm{g}} \\ &\text{(ii)}\quad \nabla\times\mathscr{I} = \boldsymbol{j}_{\mathrm{g}}-\alpha_0\frac{\partial\mathscr{G}}{\partial t} &&\text{(iv)}\quad \nabla\cdot\mathscr{I} = 0 \end{aligned} \tag{1}$$

and the momentum density $\boldsymbol{\pi}$, associated with the body, can be identified as

$$\boldsymbol{\pi} = \nabla\times\mathscr{I}+\alpha_0\frac{\partial\mathscr{G}}{\partial t}. \tag{2}$$

In the above equation, ρ_{g} is the gravitational mass density, $\boldsymbol{j}_{\mathrm{g}}$ is the mass current density and α_0 and δ_0 are, respectively, the gravitational permittivity and the inertial permeability of free space.

The gravito-inertial field may also be expressed in terms of a scalar potential ψ_{g} and a vector potential $\boldsymbol{\Psi}_{\mathrm{i}}$:

$$\text{(i)}\quad \mathscr{G} = -\nabla\psi_{\mathrm{g}}+\frac{\partial\boldsymbol{\Psi}_{\mathrm{i}}}{\partial t} \qquad \text{(ii)}\quad \boldsymbol{I} = \delta_0\mathscr{I} = \nabla\times\boldsymbol{\Psi}_{\mathrm{i}}. \tag{3}$$

For a body having a mass m_0, measured in its own rest frame, and moving with a velocity $\boldsymbol{u}$ relative to the observer, the retarded potentials ψ_{g} and $\boldsymbol{\Psi}_{\mathrm{i}}$ are given by

$$\text{(i)}\quad \psi_{\mathrm{g}} = -\frac{1}{4\pi\alpha_0}m_0\frac{1}{r-\boldsymbol{r}\cdot\boldsymbol{u}/c'} \qquad \text{(ii)}\quad \boldsymbol{\Psi}_{\mathrm{i}} = \frac{\delta_0}{4\pi}m_0\frac{\boldsymbol{u}}{r-\boldsymbol{r}\cdot\boldsymbol{u}/c'} \tag{4}$$

where $\boldsymbol{r}$ is the position vector of the observer.

The mass variation with speed, deduced by an observer and predicted by the special theory of relativity, was shown to arise purely from the propagation of the gravito-inertial field. The field equations impose an upper limit on the speed of the body, equal to the propagation speed $c' = (\alpha_0\delta_0)^{-1/2}$ of the gravito-inertial field and further considerations show that c' must be equal to c.

The present paper is concerned primarily with the energy–momentum relations of the gravito-inertial field. It will be demonstrated that the Newtonian laws of motion for a slowly moving particle arise directly from the field equations (1). This treatment, when extended to the case of any velocity, leads to a completely consistent, and effectively relativistic, theory of a neutral elementary particle and its dynamics.

2. Gravito-inertial field energy

By the well-known procedures employed in the analogous electromagnetic situation, we obtain for the energy densities of the gravitic and inertial field, respectively:

$$\text{(i)}\quad \mathcal{U}_g \equiv \tfrac{1}{2}\alpha_0\,\mathcal{G}^2 \qquad \text{(ii)}\quad \mathcal{U}_i \equiv \tfrac{1}{2}\delta_0\mathcal{I}^2. \tag{5}$$

We can also identify the gravito-inertial equivalent of the electromagnetic Poynting vector as

$$\mathcal{N} = -(\mathcal{G}\times\mathcal{I}). \tag{6}$$

For a particle at rest the only contribution to its gravito-inertial field energy is that due to the gravitic field.

The following analysis is restricted to an isolated (and neutral) elementary particle. The calculation is similar to that given by Abraham (see, e.g., Abraham 1920) for the electrostatic self-energy of an electron. We shall assume here a spherically symmetric, but otherwise completely general, *volume* distribution, $\rho(r)$, for the gravitational mass density, extending to a distance R.

With the aid of equations (4(i)) and (3(i)), the total gravitic field energy U_g is then given by

$$U_g = 2\pi\alpha_0\left[\int_{r=0}^{R}\frac{1}{r^2}\left\{\int_{r'=0}^{r} r'^2\rho(r')\,dr'\right\}^2 dr + \int_{R}^{\infty}\frac{1}{r^2}\left\{\int_{r'=0}^{R} r'^2\rho(r')\,dr'\right\}^2 dr\right].$$

Integrating by parts and using the fact that $\alpha_0\delta_0 = 1/c^2$, one finds that

$$U_g = 4\pi\delta_0\left[\int_{r=0}^{R}\left\{\int_{r'=0}^{r}\rho(r')r'^2\,dr'\right\} r\rho(r)\,dr\right]c^2. \tag{7}$$

For a particle at rest the term multiplying c^2 in (7) is a function of the gravitational mass distribution. Its explicit form and its relationship to the inertial mass will be dealt with in the next section and the restriction, imposed on the behaviour of $\rho(r)$, near $r = 0$, by the convergence requirements of the integrals in (7), will be discussed in a later section.

3. Inertial reaction

When a body is accelerated, the time variation of the vector potential produces a local gravitational field in a direction opposite to the acceleration. The resultant force due to this gravitic field must be counterbalanced by an external force in order to maintain the accelerated motion. This provides an immediate insight into the fundamental nature of inertia in a body.

For a particle accelerated *from rest*, the reaction force is given by

$$\boldsymbol{F} = -\int_0^R \mathcal{G}^*(r)\rho(r)4\pi r^2\,dr \tag{8}$$

where $\mathcal{G}^*(r)$ is the induced gravitic field strength at a radial distance r and can be obtained from equations (3(i)) and (4(ii)). Hence

$$\boldsymbol{F} = -4\pi\,\delta_0\left[\int_{r=0}^{R}\left\{\int_{r'=0}^{r}\rho(r')r'^2\,\mathrm{d}r'\right\}r\rho(r)\,\mathrm{d}r\right]\dot{\boldsymbol{u}}. \tag{9}$$

The reaction force is thus directly proportional to the acceleration, in agreement with the Newtonian law of mechanics. It follows from (9) that the inertial mass of the elementary particle is given by

$$m_i = 4\pi\delta_0\int_{r=0}^{R}\left\{\int_{r'=0}^{r}\rho(r')r'^2\,\mathrm{d}r'\right\}r\rho(r)\,\mathrm{d}r. \tag{10}$$

This is precisely the factor which relates U_g to c^2 in (7). Thus, from the above analysis, we have that, for an external force $\boldsymbol{F}$,

$$\boldsymbol{F} = \frac{U_g}{c^2}\dot{\boldsymbol{u}}, \qquad U_g = m_i c^2. \tag{11}$$

It should be noted that, while U_g and m_i are structure dependent, through $\rho(r)$, the relationship between them is independent of the structure.

4. The principle of equivalence and elementary particle structure

By considering a particle in free fall and using the Galilean *reductio ad absurdum* argument, we can conclude that m_i is proportional to m_g. As was shown, however, in our previous paper, this becomes an equality

$$m_i = m_g = m. \tag{12}$$

The total rest energy of the particle, (7) and (11), is now simply equal to mc^2.

In view of (12), a clue to the structure of our fundamental particle can now be gained from equation (10) by noting that, since R is arbitrary,

$$\rho(r) = \frac{1}{\delta_0 r^2}. \tag{13}$$

It is interesting to note that (13) represents precisely the limiting variation of mass density near the origin required by the convergence condition in the integrals in equations (7) and (9).

Further, the gravitational potential energy V_g of the particle with this mass density distribution is such that $-V_g = U_g = mc^2$, that is the gravitational binding energy is identical with the rest energy.

5. The gravito-inertial field tensor

Before proceeding further with the dynamical analysis, it is convenient to cast the field equations into a covariant form. The theory of the gravito-inertial field, as shown in our previous work, leads to results which are, in all aspects, consistent with the special theory of relativity. Further, the invariance of the speed of propagation of the field shows that the field equations must be Lorentz invariant. Hence, we can introduce a metric tensor $g_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ with signature $+1, -1, -1, -1$, without thereby invoking any results of the special theory of relativity.

We now introduce the antisymmetric gravito-inertial field tensor $D^{\mu\nu}$, defined by

$$D^{\mu\nu} = \partial^\mu \Psi^{\nu} - \partial^\nu \Psi^{\mu} \tag{14}$$

where Ψ is a four-potential:

$$\Psi \equiv (\psi_0, \boldsymbol{\Psi}) = \left(-\frac{1}{c}\psi_g, \boldsymbol{\Psi}_i\right) \tag{15}$$

whose components ψ_g and $\boldsymbol{\Psi}_i$ are given by equations (4).

The components of $D^{\mu\nu}$ are given by the array

$$D^{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{c}\mathscr{G}^1 & \frac{1}{c}\mathscr{G}^2 & \frac{1}{c}\mathscr{G}^3 \\ -\frac{1}{c}\mathscr{G}^1 & 0 & -I^3 & I^2 \\ -\frac{1}{c}\mathscr{G}^2 & I^3 & 0 & -I^1 \\ -\frac{1}{c}\mathscr{G}^3 & -I^2 & I^1 & 0 \end{pmatrix} \tag{16}$$

6. The energy–momentum relations

In order to obtain the energy and momentum relations for non-zero velocities it is necessary to include the contributions due to the inertial field.

The exact expressions required can be obtained most easily from the gravito-inertial energy–momentum tensor T which can be generated in the usual manner. Thus

$$D_{\mu\nu} j^{\nu} = -\partial_{\lambda} T_{\mu}{}^{\lambda} \tag{17}$$

where $j^{\nu} \equiv (j_0, \boldsymbol{j}) = (c\rho_{\mathrm{g}}, \boldsymbol{j}_{\mathrm{g}})$ is the four-current density.

Substitution from (16) into equation (17) then yields

$$T^{\mu\nu} = \left(\begin{array}{c|c} \mathscr{U} & -\frac{1}{c}\mathscr{N} \\ \hline \frac{1}{c}\mathscr{N} & \alpha_0 \boldsymbol{\mathscr{G}}\boldsymbol{\mathscr{G}} + \delta_0 \boldsymbol{\mathscr{I}}\boldsymbol{\mathscr{I}} - \mathscr{U}\mathbf{1} \end{array}\right) \tag{18}$$

where $\mathscr{U} = \frac{1}{2}\alpha_0 \mathscr{G}^2 + \frac{1}{2}\delta_0 \mathscr{I}^2$ is the energy density of the field and $\mathscr{N}$ is the gravito-inertial Poynting vector defined in equation (6).

Following the treatment given by Rohrlich (1960) for the analogous problem in the classical theory of the electron, it is now possible to identify the energy and momentum due to the field of the particle as

$$\text{(i)} \qquad W = \gamma^2 \int \mathscr{U}\, \mathrm{d}^3x - \frac{\gamma^2}{c} \int \mathscr{N} \cdot \boldsymbol{u}\, \mathrm{d}^3x$$

$$\text{(ii)} \qquad \boldsymbol{P} = \frac{\gamma^2}{c^2} \int \mathscr{N}\, \mathrm{d}^3x + \frac{\gamma^2}{c^2} \int \mathbf{T} \cdot \boldsymbol{u}\, \mathrm{d}^3x \tag{19}$$

where

$$\mathbf{T} = \alpha_0 \boldsymbol{\mathscr{G}}\boldsymbol{\mathscr{G}} + \delta_0 \boldsymbol{\mathscr{I}}\boldsymbol{\mathscr{I}} - \mathscr{U}\mathbf{1}$$

and

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}.$$

For a particle in uniform motion

$$\boldsymbol{\mathscr{I}} = -\alpha_0 (\boldsymbol{u} \times \boldsymbol{\mathscr{G}})$$

so that, using the same mass density distribution function $\rho(r)$ as before, and on substituting equation (12), the momentum associated with the particle, given by equation (19), becomes

$$\boldsymbol{P} = \frac{U_{\mathrm{g}}}{c^2} \boldsymbol{u} = m_{\mathrm{i}} \boldsymbol{u}. \tag{20}$$

Hence the theory gives a low-velocity momentum identical with the normal Newtonian concept. For a relativistic particle, the complete expressions (19) yield (following Rohrlich) the appropriate correct results:

$$W = mc^2, \qquad \boldsymbol{P} = m\boldsymbol{u} \tag{21}$$

where, however,

$$m = m_0 \left(1 - \frac{u^2}{c^2}\right)^{-1/2}.$$

A treatment similar to the Abraham–Lorentz theory would yield the low-velocity momentum and energy:

$$\text{(i)} \qquad \boldsymbol{P} = \frac{1}{c^2}\int \mathcal{N}\,\mathrm{d}^3X = \tfrac{4}{3}m_1\boldsymbol{u}$$

$$\text{(ii)} \qquad W = \int \mathcal{U}\,\mathrm{d}^3X = m_1c^2. \tag{22}$$

The inconsistencies between the relationships (22(i)) and (22(ii)) were removed by Rohrlich (1960) for the case of the electron, through a proper definition of momentum via the energy–momentum stress tensor. This latter approach not only avoids the undesirable 4/3 factor in (22(i)), but also ensures the Lorentz invariance of the energy and momentum.

For a free gravito-inertial field, the momentum density is given by

$$\boldsymbol{\pi} = -\frac{1}{c^2}(\mathcal{G}\times\mathcal{I}). \tag{23}$$

In the presence of a moving field source, however, such as in the case of the particle considered above, the additional terms due to the gravito-inertial stresses must be included. The latter are also connected with the problem of the stability of the particle, which otherwise would disappear through a gravitational collapse. As Rohrlich has pointed out, whenever the self-interaction can be separated from external interactions in a relativistically invariant manner, which is true for the definitions (19), instability problems can be removed by a renormalization.

7. Conclusions

It has been shown that a consistent description of a neutral elementary particle can be constructed on the basis of the gravito-inertial field theory.

The dynamics and, in particular, the very nature of the inertial mass of the particle are completely described in terms of the gravito-inertial field. The latter itself is specified solely in terms of the concept of gravitational mass.

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